The Mathematics of Vinegar Volcanoes: modeling an endothermic reaction with Linear and Exponential Functions at Al Wakra Boys' Independent Secondary School

Students should work in pairs or threes.

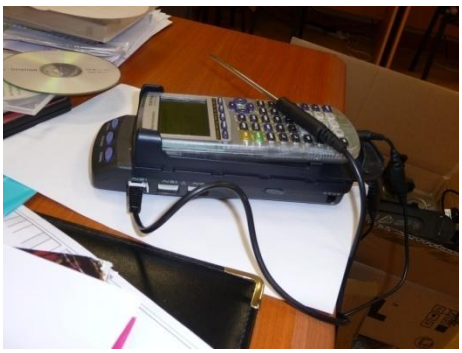
Demonstrate the reaction when approximately ½ teaspoon of bicarbonate is added to 50mL of vinegar (baking soda is safest but ammonium bicarbonate will also work).

Caution: bicarbonate can damage the eyes – ensure safety precautions in class and make sure the students wash their hands if they touch the bicarbonate or the vinegar.

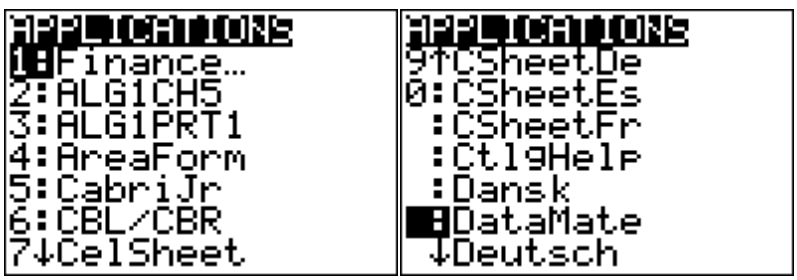
Ask the students to predict what is happening to the temperature as the mixture fizzes. Most students will predict the temperature increases.

Invite the students to investigate and to form a mathematical model of the temperature/time function. (At Grades 10 and 11 they will use a linear model, at Grade 12 they can be introduced to the exponential model).

Connect the CBR data logger to the TI 83 or TI 84 calculator and turn it on. Make sure the calculator has the appropriate DataMate application (for information on this consult the CBL manual).

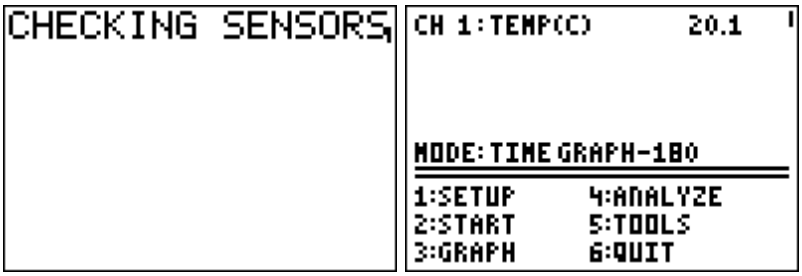


Plug a temperature probe into Channel 1.



Press APPS and scroll down ▼ to the **DataMate** application. Press ENTER.

The **DataMate** application will start. It may take a moment to identify that there is a temperature probe plugged in. The temperature is displayed on the calculator screen.



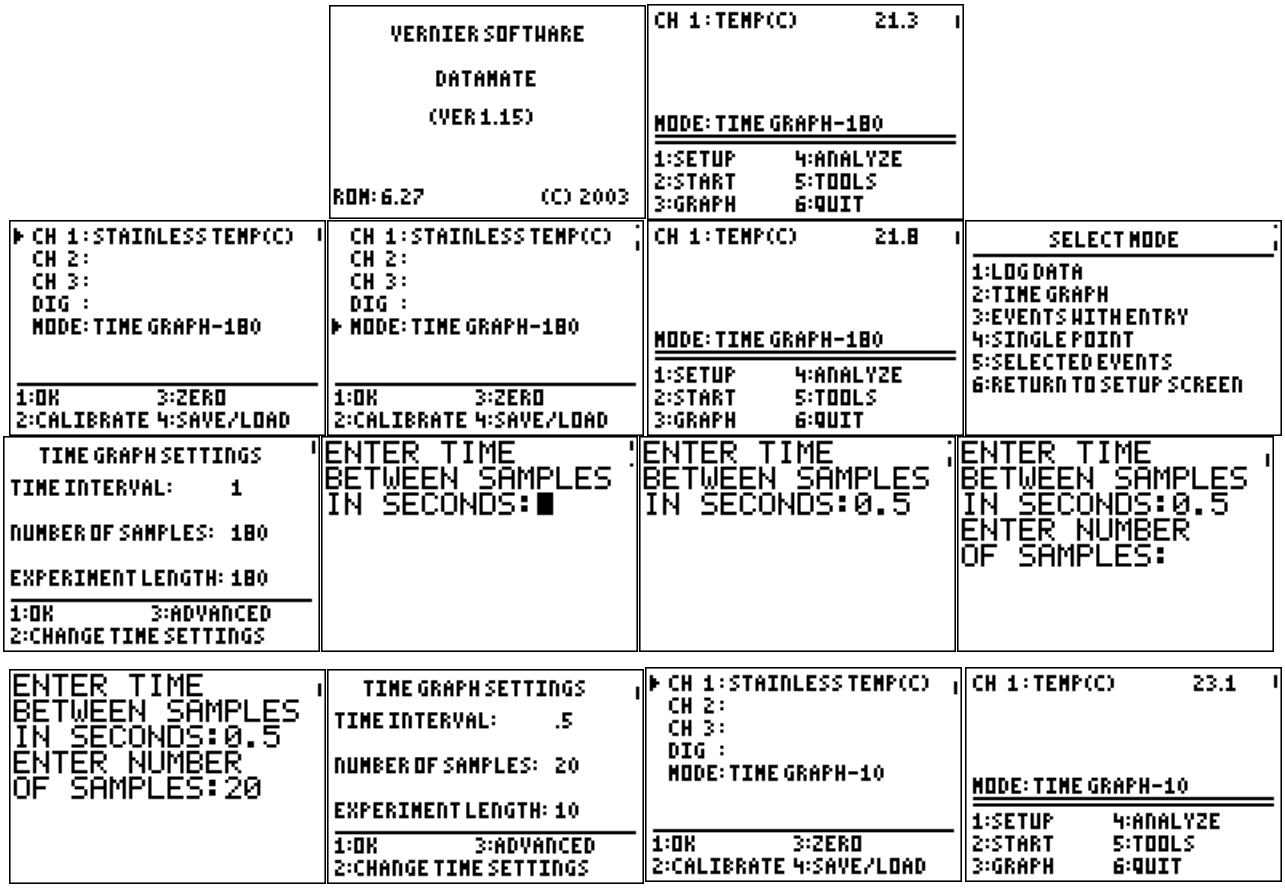
Setting up the data logger.

Set the data logger to record the temperature every 0.5 second for 10 seconds.

To do this press **1: Setup** then scroll down to ▼ to **Mode: Time Graph** and press ENTER.

Press: **2: TIME GRAPH** then **2:CHANGE TME SETTINGS**.

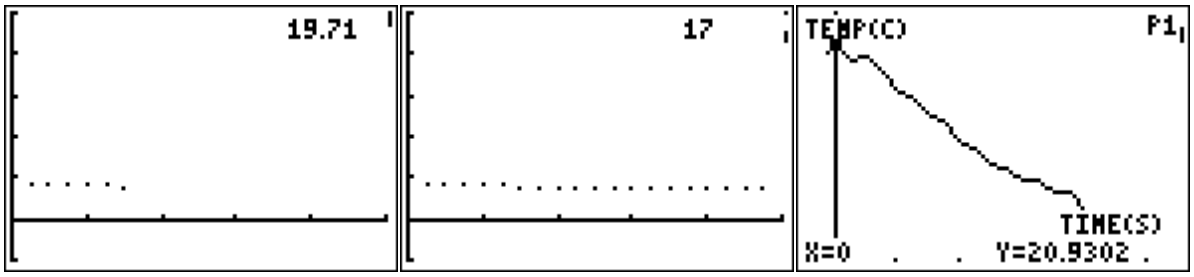
Type **0.5** ENTER then **20** ENTER to set the time interval. Then press **1:OK** and **1:OK** again to return to the main screen.



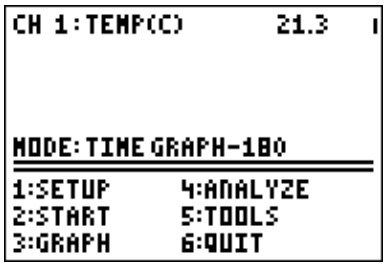
The experiment

Pour approximately 50mL of vinegar into a plastic cup. Place the thermometer in the cup and note the temperature. Add ½ teaspoon of bicarbonate and press [ENTER].

The CBL will record the temperature for 10 seconds, drawing the graph in "real time". DataMate then will then automatically rescale the graph.

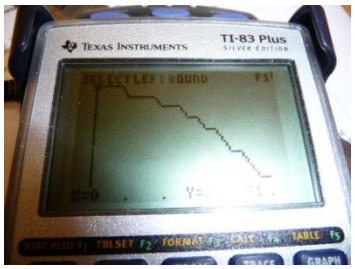


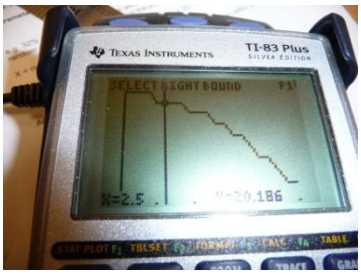
Press [ENTER] to return to the **DataMate** main screen.



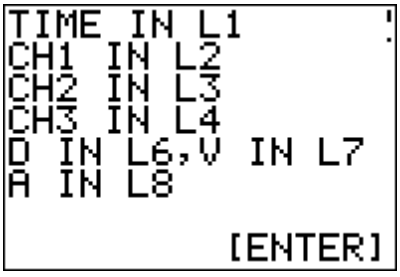
To select a region of the graph for analysis press [3] to see the graph, then [ENTER]to obtain the **SELECT REGION** option.

Press [2] then move the cursor to the left bound using the [▶] key and press [ENTER]. Use the [▶] key to move the cursor to the right of the desired region and press [ENTER] . DataMate will graph the desired region.



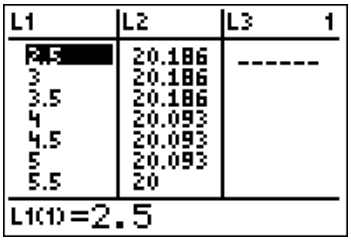


Press then [1] to return to the main screen. Press [6] to quit the programme. The data is stored in the calculator statistics lists **L1** and **L2**, thus the calculator can be disconnected from the CBL. Press [ENTER] to clear the screen

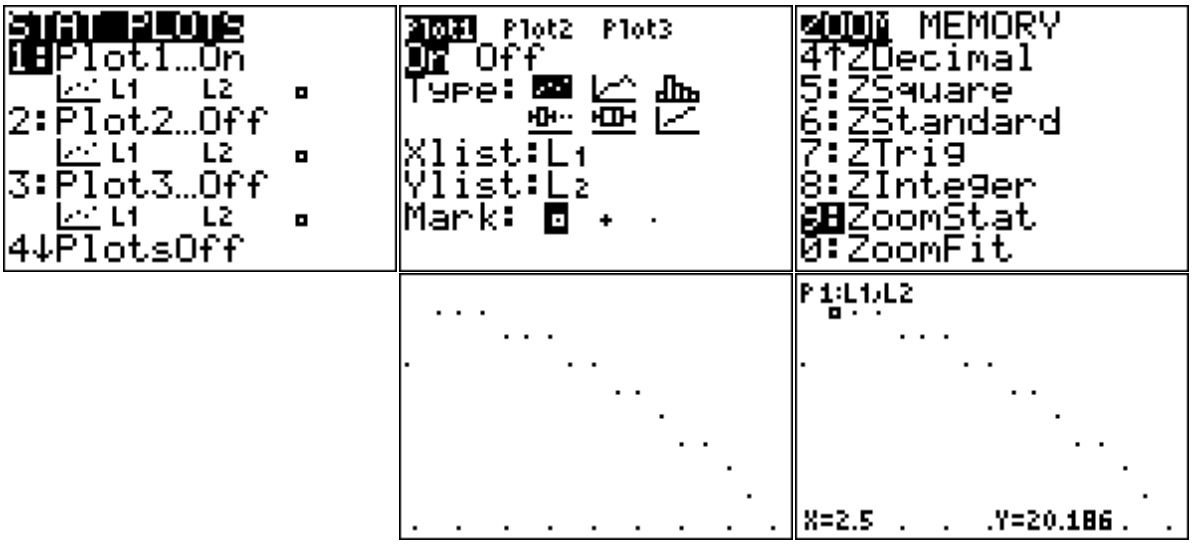


Doing the mathematics

To see the data in the lists press [STAT] then [1] or [ENTER].

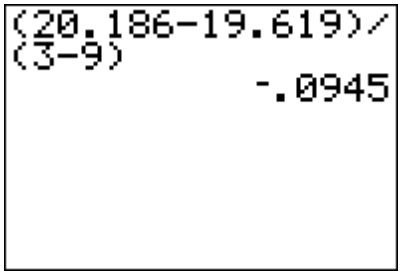


To draw the data as a dot plot press [2nd][Y=] to turn the statistics plot menu on. Set the type to a dot plot then press [ZOOM][9] for Zoom Statistics. Press [TRACE] and the [▶] arrow keys to read the data.

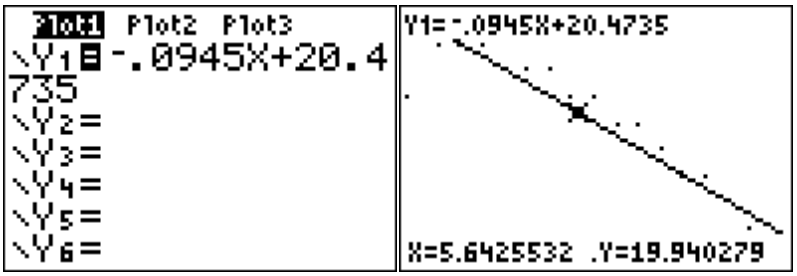


Forming the mathematical model

Use [TRACE] and the [▶] arrow keys to read through the data and select points to form the equation for the linear model.



Press [Y=] to open the function editor and type in the linear equation. Press [GRAPH] and both graphs appear simultaneously. Use the arrow keys [▼] to move between the two graphs and [▶] to read values.



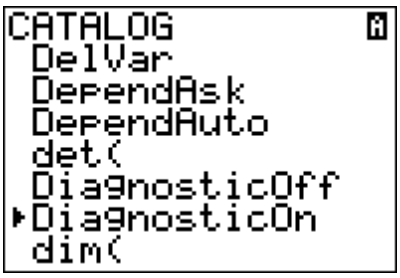
Questions to explore:

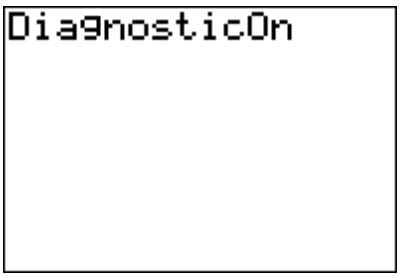
- How well does your linear model predict the temperature after 5.5 seconds, compared to the data from the statistics editor or scatter plot?
- What does your linear model predict the temperature will be after 65 seconds? Comment on the significance of the domain
- Describe what you think happens to the temperature in the long run
- Investigate **endothermic reactions** and describe how they work.

The Exponential model.

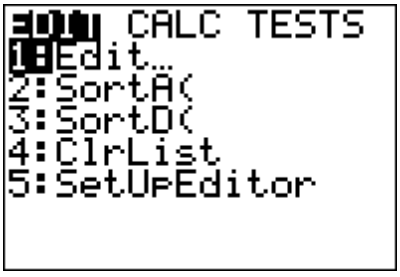
Either use by hand methods or the graphic calculator to determine whether an exponential model is a better fit.

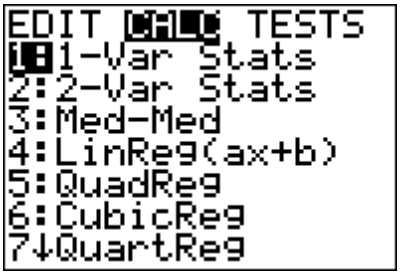
To see the **residual** the turn the **diagnostics on**. To do this press 2nd 0 **CATALOGUE.** Then scroll down ▼ and press ENTER at **Diagnostic On**. This returns you to the home screen where you can press ENTER to clear the screen and work with the diagnostic tool turned on.

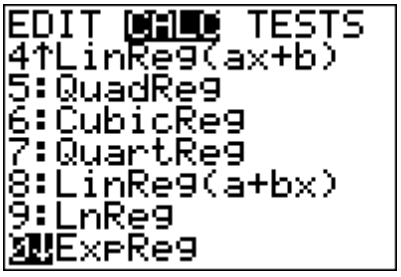


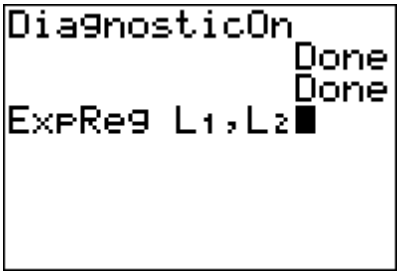


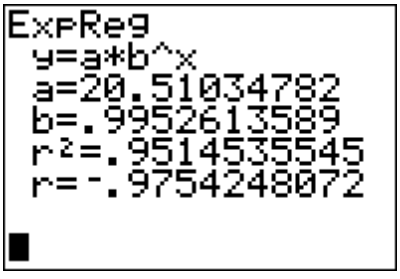
To determine the exponential equation and its residual, press STAT to open the statistics menu, then use the **Calculate** menu and choose **0: Exponential Regression** 2nd 1 **L1** , 2nd 2 **L2** and the coefficients and residual are shown.



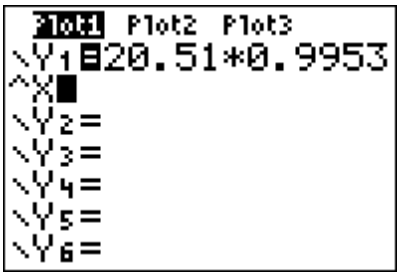


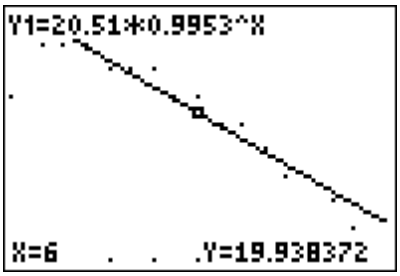






Go to the [Y=] equation editor, type in the equation and press [ENTER] to see the exponential graph displayed over the scatter plot.
Use the arrow keys [▼][▶] to move between the graphs and to read values.





Questions to explore:

- Explain the significance of the residual $\mathbf{r^2}$ and how it is calculated
- Use your calculator to determine whether a polynomial would be a better model than an exponential.